System Identification

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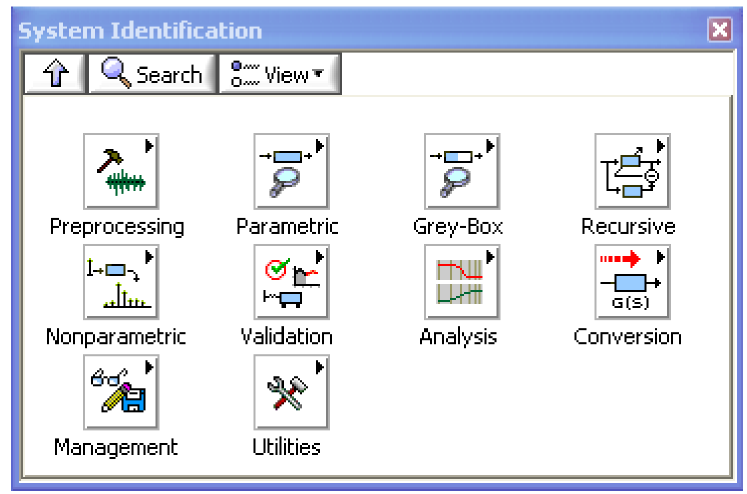
Overview

- What is System Identification?
- Where do you start?
- Exciting the System
- Data Collection
- Data Post-treatment
- Transient Analysis
- Methods of System Identification
- System Identification with LabVIEW



What is system identification?

- System Identification –My definition
  - Is a collection of tools to help you better understand the system that you are working with.
- System Identification – Official Definition:
  - To build and compliment models from measurements.
- Used for Parametric and structural Identification

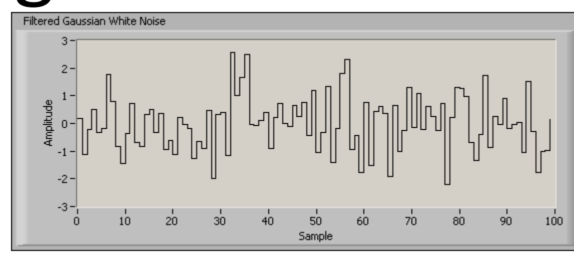
$$m\ddot{x} + b\dot{x} + kx = F$$

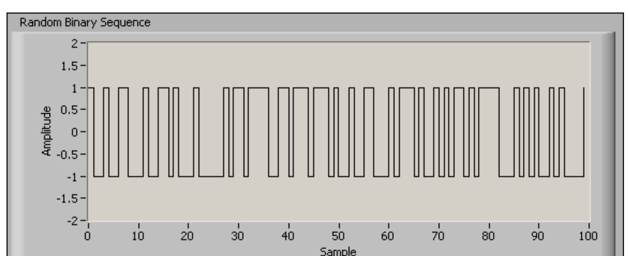
Where do you start?

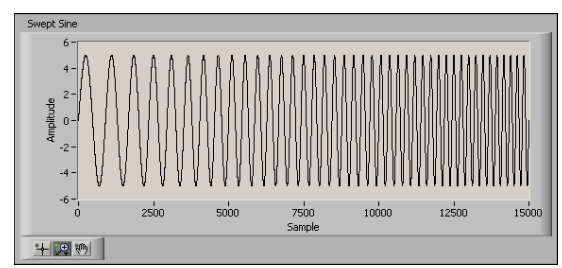
- By exploiting what you know already
  - e.g. I have a second order system, the mass is 3 kg
- Determining what information you need
  - e.g. I need to know the systems bandwidth
- Exciting the System
- Collecting data to get that information
  - e.g. past input output data to construct a Bode plot

Exciting the System

- Goal is to excite all the frequencies of interest
  - In theory, an impulse to a system does exactly this
  - Typically not practical (e.g. online systems)
- Excitation Options (all high bandwidth inputs)
  - Filtered Gaussian Noise
  - Random Binary Signal
  - Pseudo Random Binary Sequence (PRBS)
  - Chirp

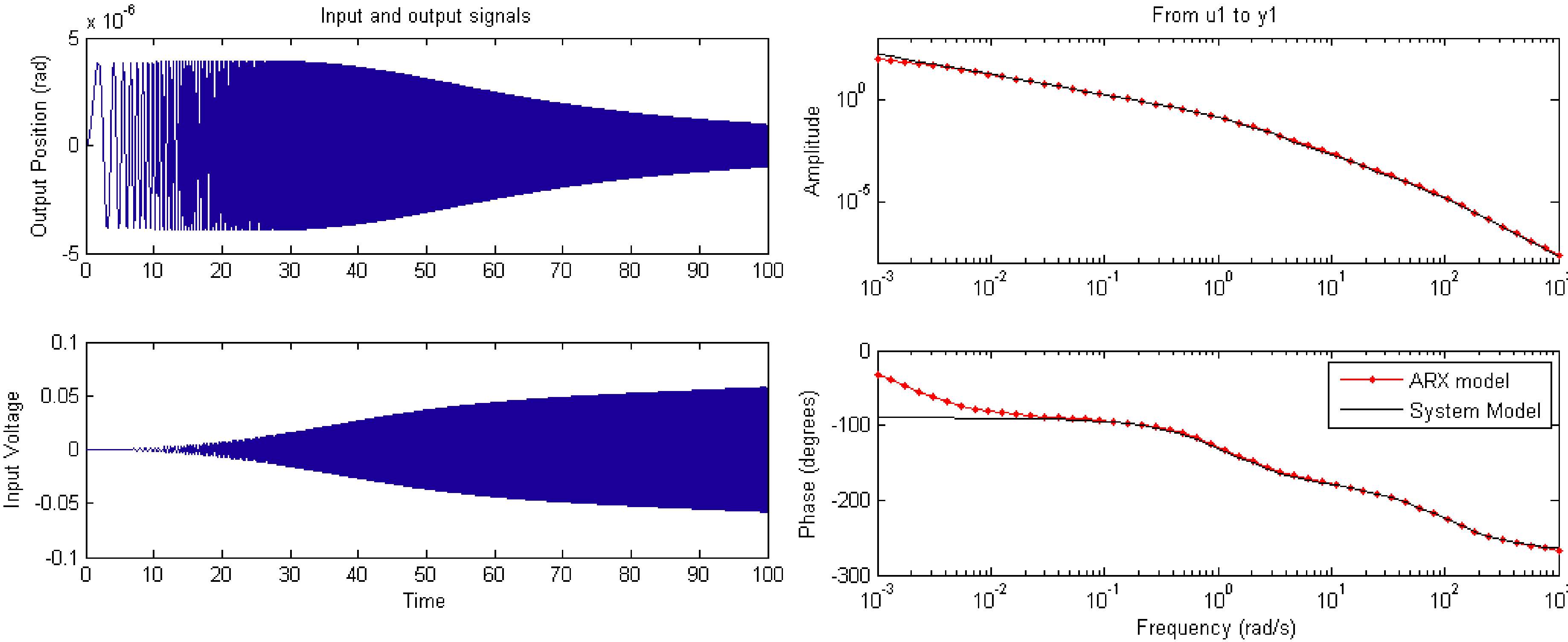






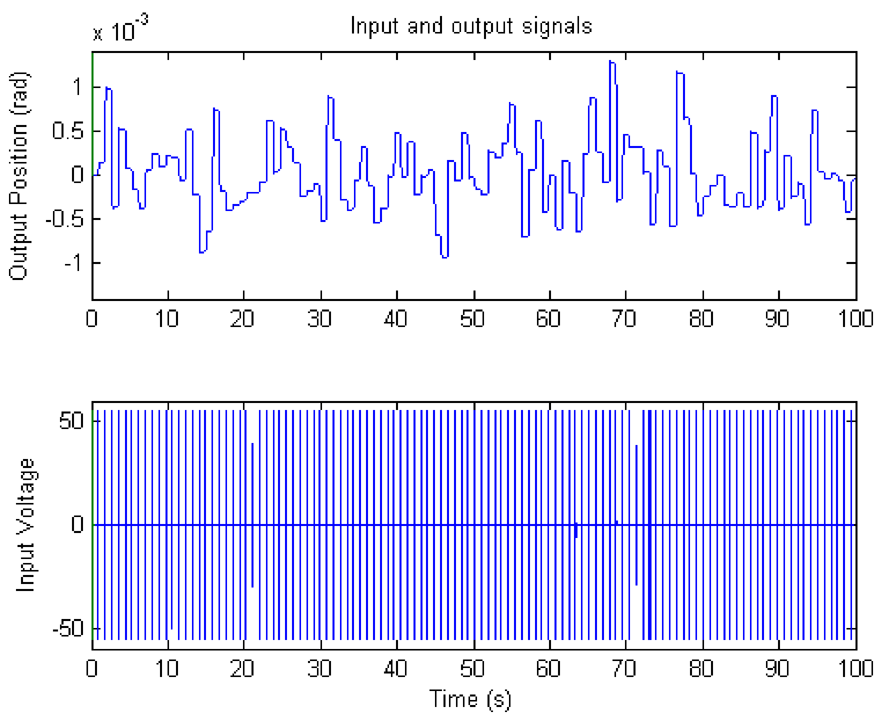
Exciting the System…

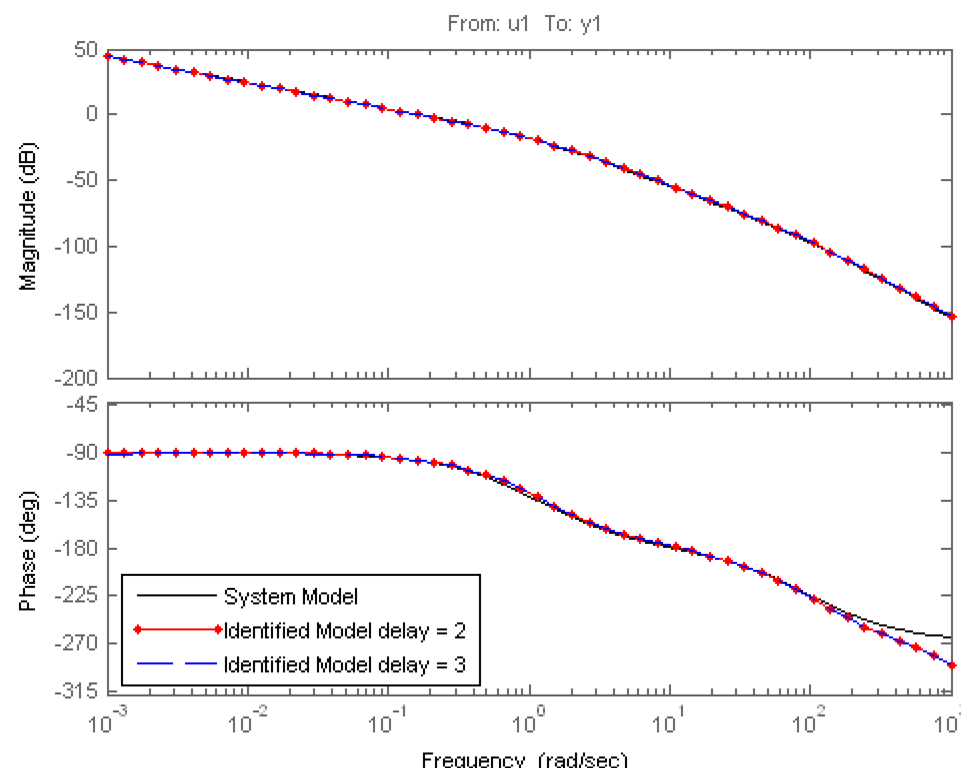
- What is bad about a chirp (swept sine) signal?
- Is this a problem? What do you recommend?



Exciting the System…

- White Noise Excitation
- High and low bandwidth excitation!



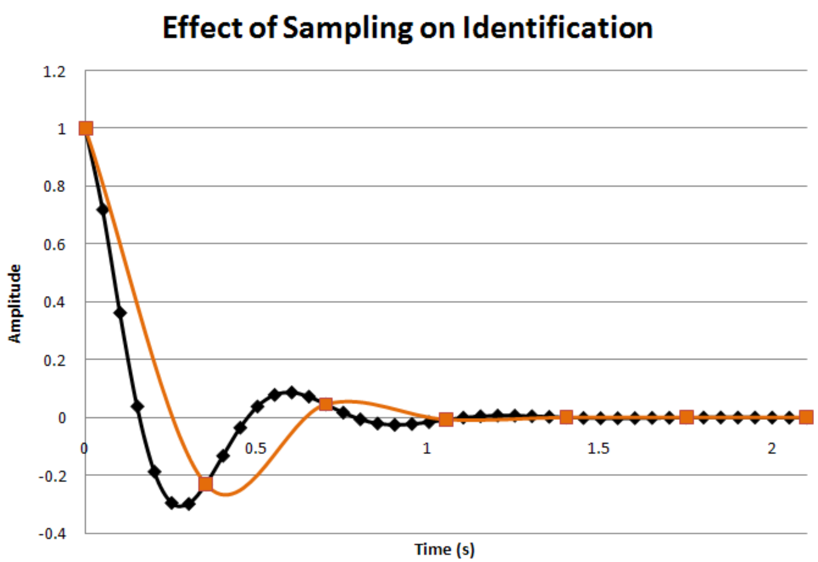


Data Collection

- Data collection is the first and most important step in system identification
  - Data collection is a design problem
  - The data you collect depends on what your identification objectives are
    - e.g. temperature dynamics in a room
    - e.g. stability of a motor vehicle
- Goal: to get as much of the system's frequency content as possible
  - In the 1950's the had a game show called "Name that tune" where the quicker you got the song from a few notes the better you did.
  - With System Identification you want the whole song (usually).

Data Collection…

- Rule of thumb: System time constant dictates sampling rate
  - This should be at least 10x the time constant so that you have 5-8 samples over the rise time
- Pre-Filtering (done before sampling, analog)
  - Anti-Aliasing filter at ½ the sampling frequency
    - If the data was sampled at 1kHz then filter all signals above 500Hz.



Orange -- Sampled every 0.35s (2.85 Hz)
Black -- Sampled every 0.05s (20 Hz)

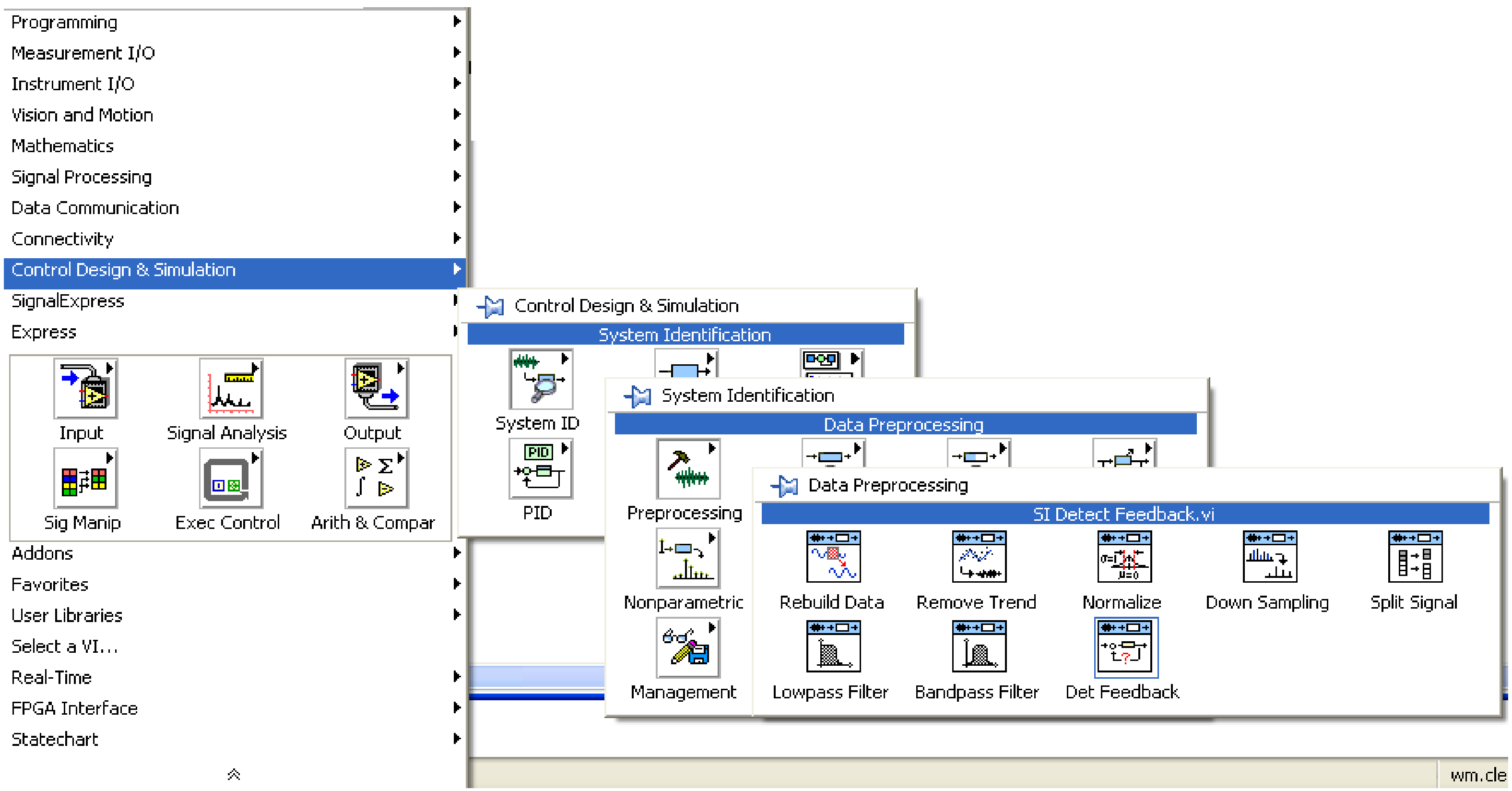
Identified System Dynamics Can Significantly Change

Data Post-Treatment

- Filtering
  - Band Enhancement
    - e.g. filtering below and above a known resonance to accurately capture i
  - Noise removal
- Removal of Drift
  - Sensor drift due to temperature fluctuations
- Separate Experimental and Validation Data
- Removal of means (makes algorithms more sensitive)
- After you have the data, and you are confident that it has the frequency content of interest, you are ready to begin using the System Identification tools.

Data Post-Treatment…LabVIEW

- These toolkits are located in…

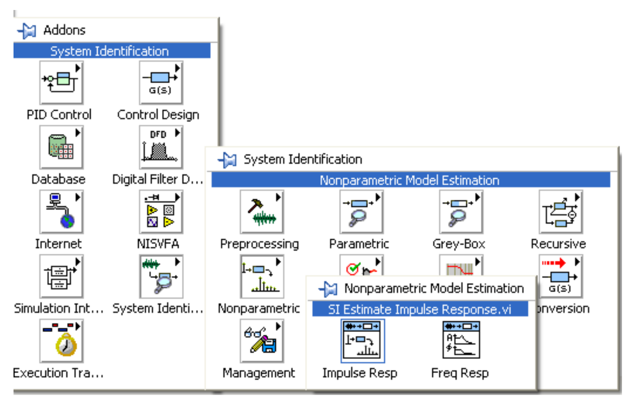


Identification Methods

- Transient Observation (knowledge)
  - Discussed last week (step/impulse response observations)
- Non-Parametric Identification (graph output)
  - Sinusoidal Transfer Function Estimate (historically used)
  - Correlation Analysis
  - Spectral Analysis
- Parametric Identification (model output)
  - Black Box Methods (unknown parameters and model)
  - Grey Box Methods (Partially known parameters/model)

Non-Parametric Identification

- Correlation analysis
  - Used to estimate Impulse response of a system without having to put an impulse into the system
  - Impulse response theoretically has all the characteristics of the system embedded in it
  - LabVIEW uses correlation analysis to do this. In order for this to work, the input and the disturbance should not be correlated.
    - This means that the input must be white with a spectral density equally distributed across the whole spectrum
- This method helps to determine:
  - The delay between the input and output
  - The system's dominant time constant
  - The system's damping
  - If there is feedback in the signals



- Drawbacks
  - This method does not result in a model that can be used for control
  - Does not work under output feedback

Non-Parametric Identification…

- Correlation Analysis Examples

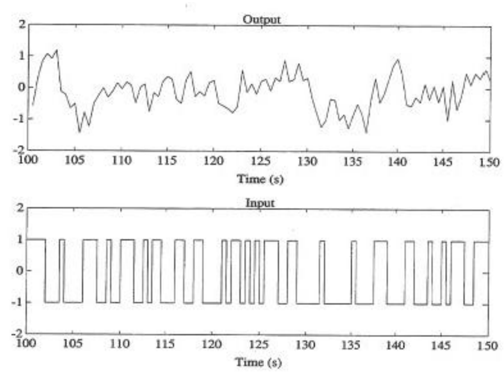
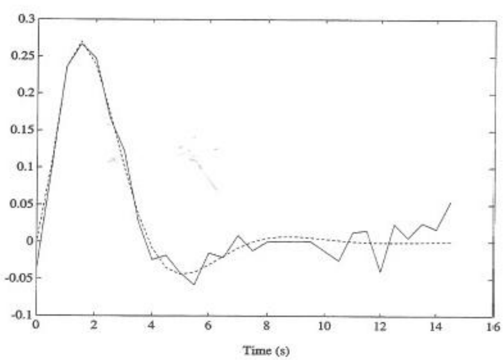
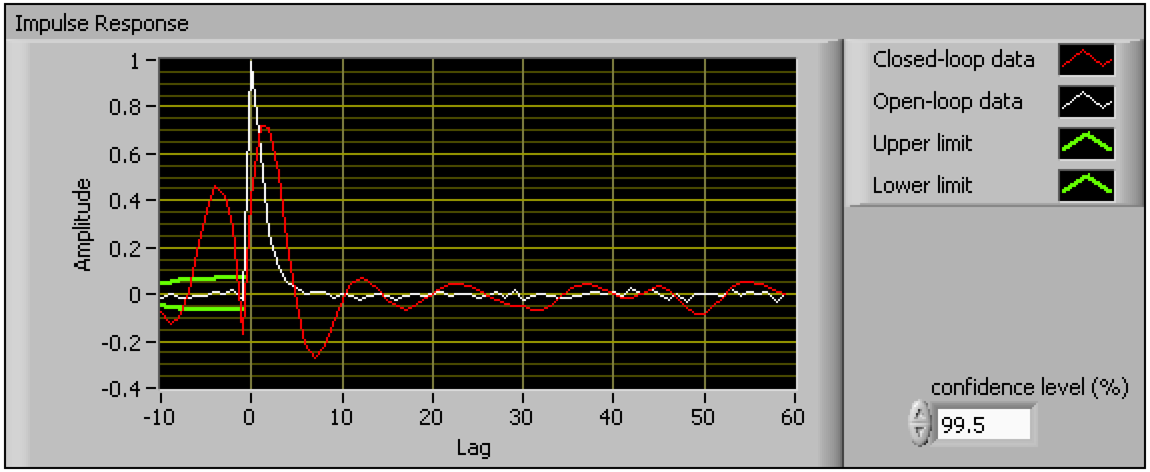


Figure 8.3: Input–output data from the system (8.10).





Closed Loop Data Doesn't Work!

Large values for negative lag imply feedback in the data

Non-Parametric Identification…

- Spectral Analysis
  - Mathematically the same as the frequency response of the impulse response
  - In practice, very different due to the differences in the estimation methods
  - Very commonly used
- This method is useful to
  - Get a **GOOD** estimate of a systems frequency response
    - Windows are usually applied for the same reasons that we use them for DFT's
    - Rule of Thumb for Lag windows: 5-10% of data length
  - To identify necessary filters before using other (parametric) methods.
  - For comparison of a the frequency response of a parametric model (from identification…i.e. did I miss something?)
- Drawbacks
  - Does not work for systems under closed loop (output feedback)
  - Does not result in a model

Parametric Methods

- Before we begin, we need to know how to relate a continuous equation to a discrete one:

$\frac{\omega(s)}{I(s)} = \frac{K_t}{J_L s + b}$	Transfer Function
$\dot{\omega} J_L + b\omega = K_t i(t)$	Differential Equation
$\frac{[\omega(n) - \omega(n-1)]}{\Delta t} J_L + b\omega(n-1) = K_t i(n-1)$	Forward Euler Approximation
$\omega(n) = \omega(n-1) + \frac{\Delta t}{J_L}[-b\omega(n-1) + K_t i(n-1)]$	Forward Euler Approximation
$\omega(n) = \omega(n-1)\left(1 - \frac{b}{J_L}\Delta t\right) + \Delta t \frac{K_t}{J_L} i(n-1)$	Forward Euler Approximation
$\omega(n) = \omega(n-1)a + i(n-1)b$	Difference Equation
$\omega(z) - z^{-1}\omega(z)a = z^{-1}i(z)b$	Equivalent Representation in z
$\frac{\omega(z)}{i(z)} = \frac{bz^{-1}}{1 - z^{-1}a}$	Transfer Function in z

Parametric Methods…

- Alternatively

$$\frac{\omega(s)}{\mathrm{I}(s)} = \frac{K_t}{J_L s + b}$$

$$\dot{\omega} J_L + b\omega = K_t i(t)$$

Main Point

$$\omega(n) = \omega(n-1)a + i(n-1)b$$

$$\omega(z) - z^{-1}\omega(z)a = z^{-1}i(z)b$$

$$\omega(t) = \int_0^t \left(-\frac{b}{J_L}\omega(\tau) + \frac{1}{J_L}K_t i(\tau) \right) d\tau$$

$$\omega(kT) = \int_0^{kT-T} \left(-\frac{b}{J_L}\omega(\tau) + \frac{1}{J_L}K_t i(\tau) \right) d\tau + \int_{kT-T}^{kT} \left(-\frac{b}{J_L}\omega(\tau) + \frac{1}{J_L}K_t i(\tau) \right) d\tau$$

$$\omega(kT) = \omega(kT-T) + T\left(-\frac{b}{J_L}\omega(kT-T) + \frac{1}{J_L}K_t i(kT-T) \right)$$

$$\omega(kT) = \omega(kT-T) + T\left(-\frac{b}{J_L}\omega(kT-T) \right) + T\left(\frac{1}{J_L}K_t i(kT-T) \right)$$

$$\omega(kT) = \omega(kT-T)\left(1 - \frac{b}{J_L}T\right) + T\frac{1}{J_L}K_t i(kT-T)$$

$$\omega(kT) = \omega(kT-T)a + i(kT-T)b$$

Parametric Methods…

- We have the past input and the past output.

$$\omega(n) = \omega(n-1)a + i(n-1)b$$

- Therefore, we can predict the next output:

$$\widehat{\omega}(n) = \omega(n-1)a + i(n-1)b$$

- And create a cost function of the error

$$J = [\omega(n) - \widehat{\omega}(n)]^2$$

$$J = [\omega(n) - \omega(n-1)a + i(n-1)b]^2$$

*All Parametric Identification Schemes Based on Minimizing This or some cost function

Parametric Methods…

- A General Linear Model
  - Literature Presents this:

$$y(k) = z^{-n} \underbrace{G(z^{-1}, \theta)}_{\text{Deterministic}} u(k) + \underbrace{H(z^{-1}, \theta)}_{\text{Stochastic}} e(k)$$

$$G(z^{-1}, \theta) = \frac{B(z, \theta)}{A(z, \theta)F(z, \theta)} \qquad H(z^{-1}, \theta) = \frac{C(z, \theta)}{A(z, \theta)D(z, \theta)}$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{k_a} z^{-k_a}$$

$$B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{k_b - 1} z^{-(k_b - 1)}$$

$$C(z) = 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_{k_c} z^{-k_c}$$

$$D(z) = 1 + d_1 z^{-1} + d_2 z^{-2} + \ldots + d_{k_d} z^{-k_d}$$

But recall this is what we are dealing with,

$$\omega(n) = \omega(n-1)a + i(n-1)b$$

$$\omega(z) - z^{-1}\omega(z)a = z^{-1}i(z)b$$

$$\frac{\omega(z)}{i(z)} = \frac{bz^{-1}}{1 - z^{-1}a}$$

$$G(z) = \frac{bz^{-1}}{1 - z^{-1}a}$$

$$z^{-1}x(k) = x(k-1)$$

$$z^{-2}x(k) = x(k-2)$$

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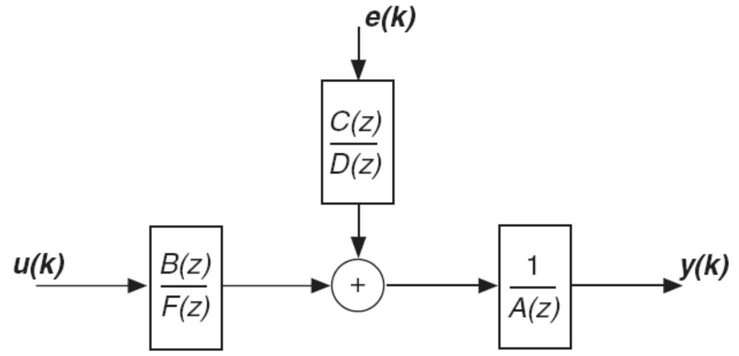
Parametric Methods…

- Ready Made Models
  - All the parametric identification methods are reduced from this
  - The reduction depends on your information about the system
  - You Choose the order of A, B, C, and D

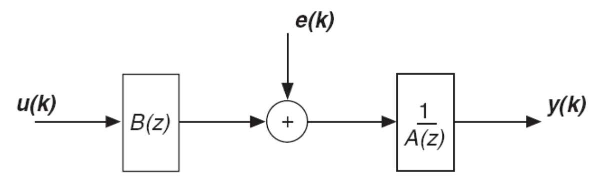
$$A(z)y(k) = \frac{z^{-n}B(z)}{F(z)}u(k) + \frac{C(z)}{D(z)}e(k)$$

$$A(z)y(k) = \frac{B(z)}{F(z)}u(k-n) + \frac{C(z)}{D(z)}e(k)$$



Parametric Methods…

- Choice of A, B, C, and D yields different models:
- ARX Model (C=D=1)



$$A(z)y(k) = z^{-n}B(z)u(k) + e(k) = B(z)u(k-n) + e(k)$$

- Many Physical Systems Described by this

$$\omega(n) = \omega(n-1)a + i(n-1)b$$

Our motor model fits in this category

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{k_a} z^{-k_a}$$

$$B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{k_b - 1} z^{-(k_b - 1)}$$

Parameters (a, b) can be determined by linear regression
It may be possible to relate these to the real system

Parametric Methods…

- First Order Model

$$\frac{\omega(s)}{\mathrm{I}(s)} = \frac{K_t}{J_L s + b}$$

ARX Identification

$$\omega(n) = \omega(n-1)a + i(n-1)b$$

$$\omega(z) - z^{-1}\omega(z)a = z^{-1}i(z)b$$

What order do I choose?

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{k_a} z^{-k_a}$$

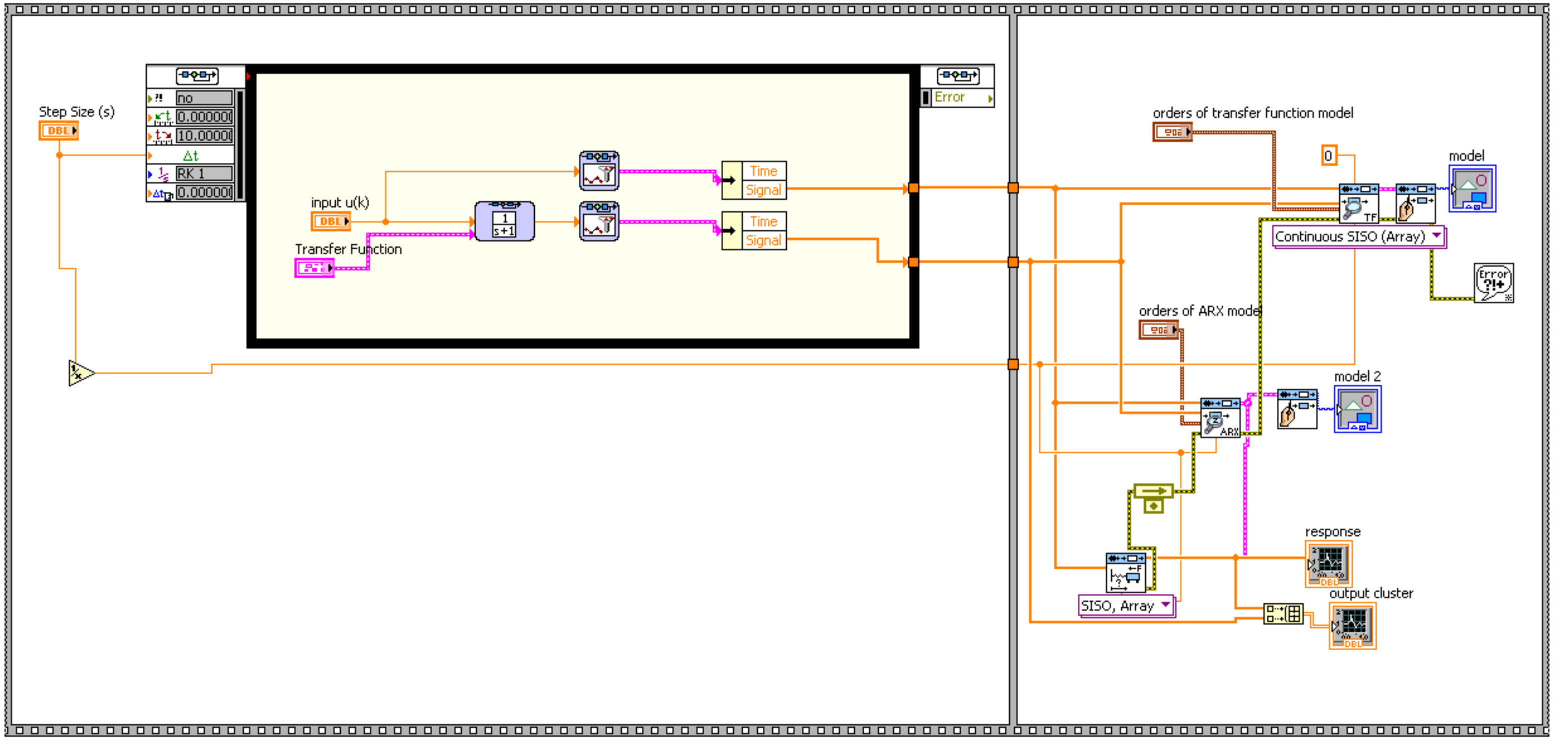
$$B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{k_b - 1} z^{-(k_b - 1)}$$

K_a and K_b are the model orders.
Therefore choose a value of K_a = 1 and K_b = 1 with a delay of 1.
Notice that this gives a discrete TF of the form,
transfer function like:

$$\frac{\omega(z)}{i(z)} = \frac{bz^{-1}}{1 - z^{-1}a}$$

Parametric Methods…LabVIEW

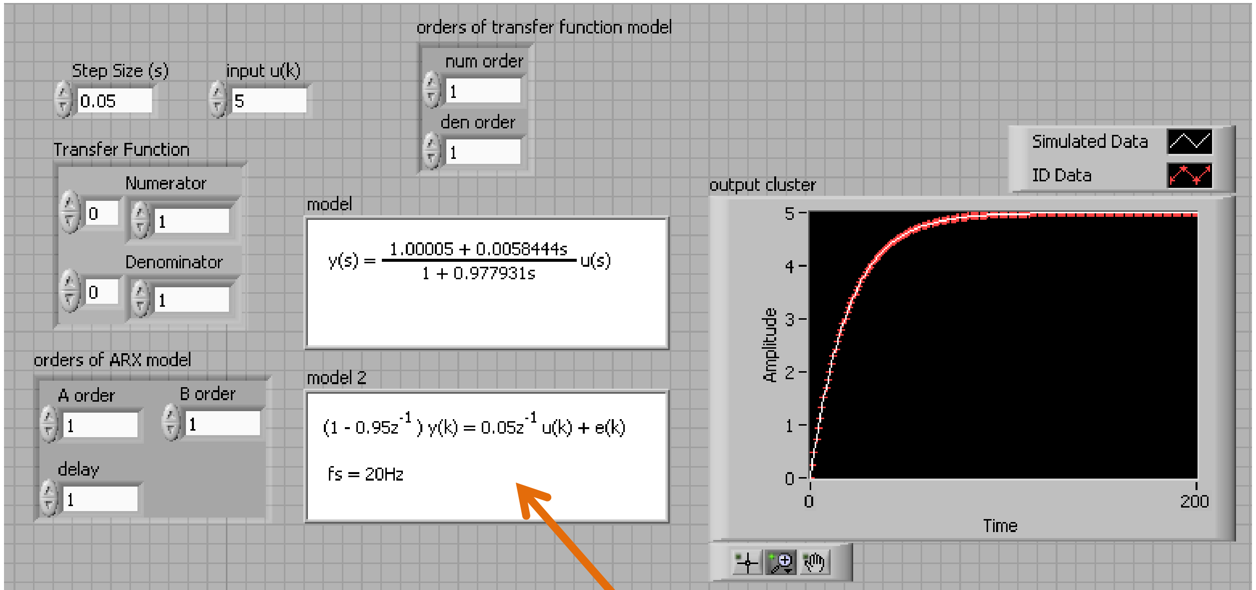
- An Example of a First order System

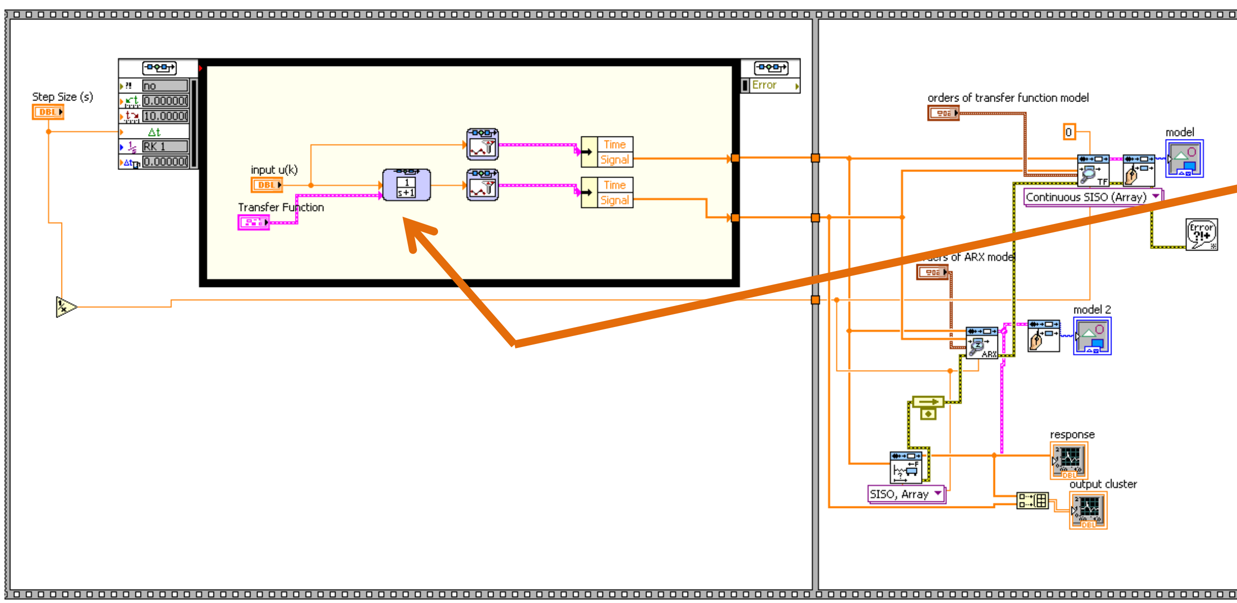


$$\frac{\omega(s)}{I(s)} = \frac{K_t}{J_L s + b}$$

$$\omega(z) - z^{-1}\omega(z)a = z^{-1}i(z)b$$

$$\frac{\omega(z)}{i(z)} = \frac{bz^{-1}}{1 - z^{-1}a}$$

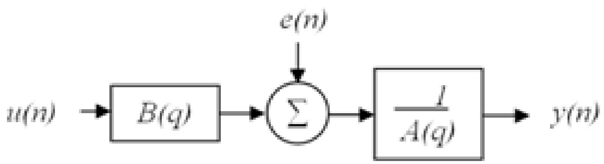
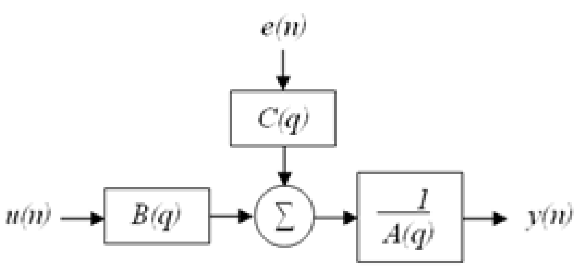
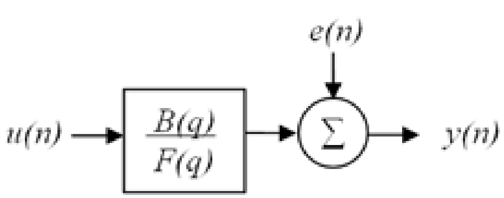
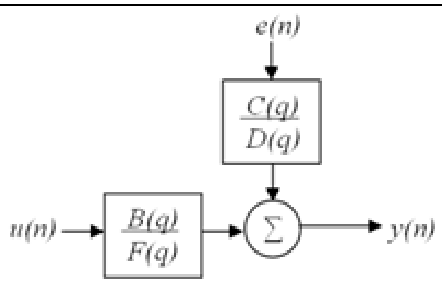




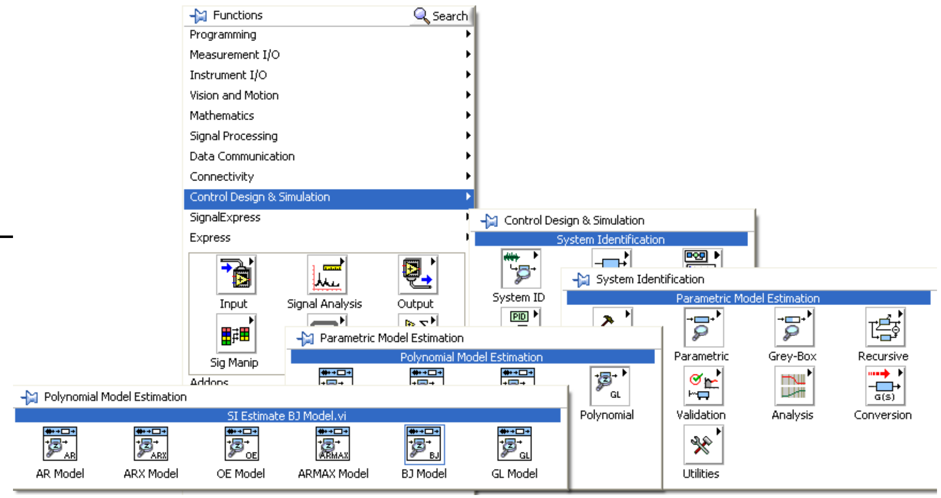
Simulated Data with $J_L = 1$, $K_t = 1$, $b = 1$

$$a = \left(1 - \frac{b}{J_L}\Delta t\right)$$

$$b = \Delta t \frac{K_t}{J_L}$$

MODEL NAME	DISTURBANCE ASSUMPTION
 ARX	Disturbances enter the process early and share the system dynamics.
 ARMAX	Disturbances enter the process early and share the system dynamics.
 Output Error (OE)	Disturbance properties are not modeled.
 Box-Jenkins (BJ)	Disturbance and system properties are independently dynamic.

Other Models



Solvers

- ARX can be solved with regression
- ARMAX, OE, BJ, and GL use
  - Gauss-Newton
- Continuous Time Options
  - Not Really continuous
  - Calculates Discrete model then uses Gauss-Newton to find coefficients.

Parametric Methods…Other

- State-Space
  - Easier to use because you only need to specify the dimension
    - This is corresponds to the number of states
  - Uses Subspace projection algorithms (principle component analysis)
    - Allows for partially known models to be used
    - Does not assume zero correlation between input and output noise (can be used when you have CL data)
    - Allows for MIMO systems
  - Drawback
    - States May not represent physical states (but can be related through similarity transform)

Parametric Methods…Other

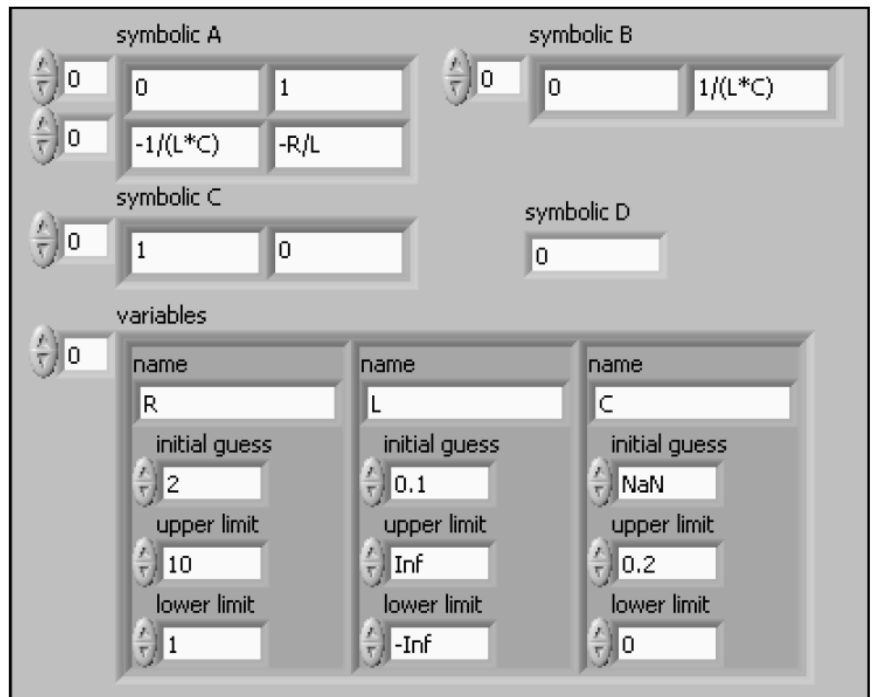
- Partially known models –RLC Ckt
- Can also be used to build TF

$$A = \begin{bmatrix} 0 & 1 \\ -1/(L \times C) & -R/L \end{bmatrix}$$

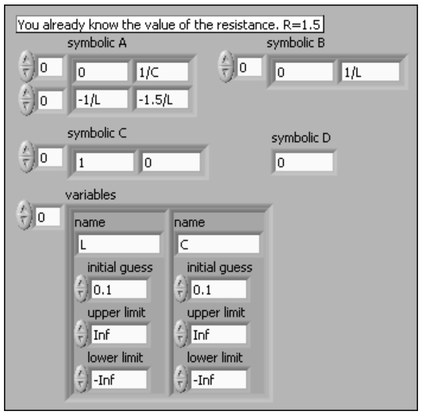
$$B = \begin{bmatrix} 0 & 1/(L \times C) \end{bmatrix}$$

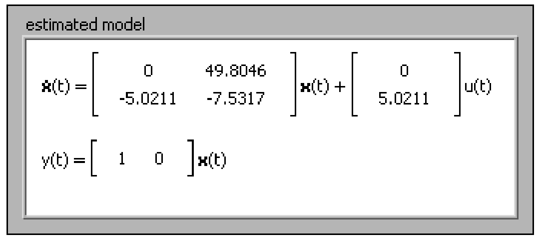
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = 0$$



Parametric Methods…Other

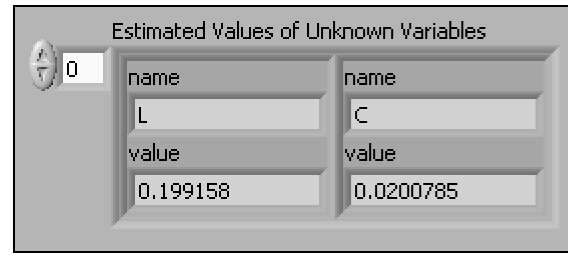


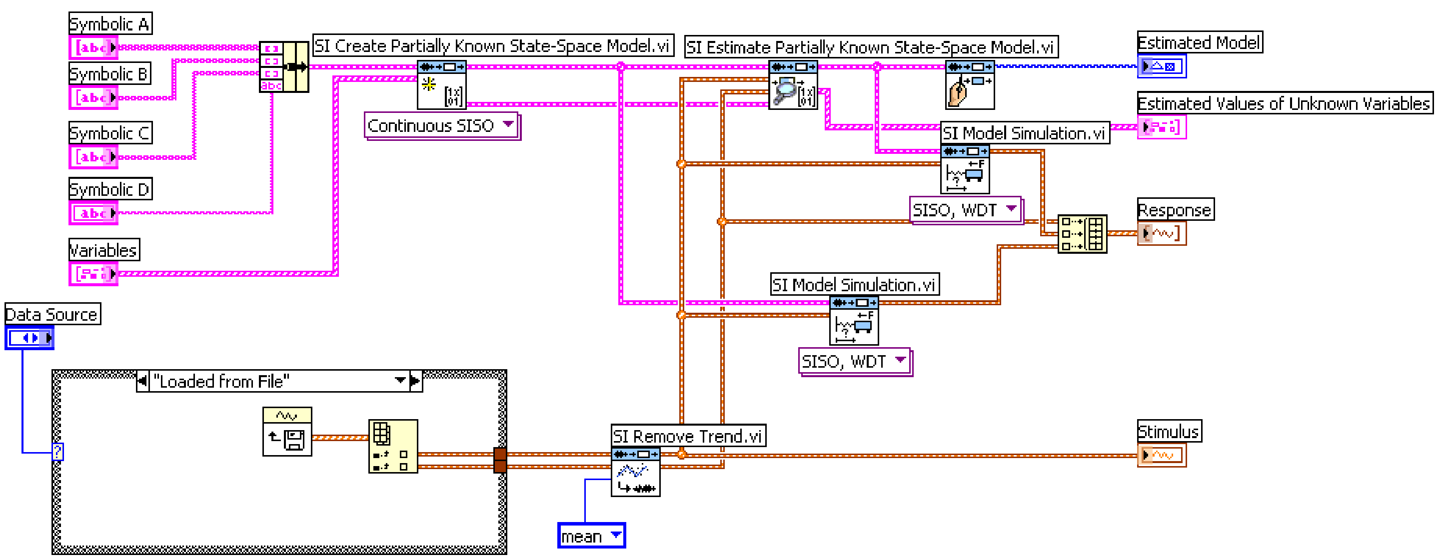


Actual Values

L = 0.2

C = 0.02

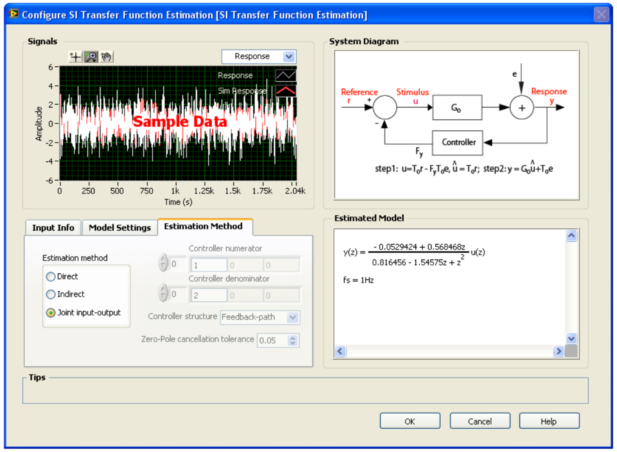


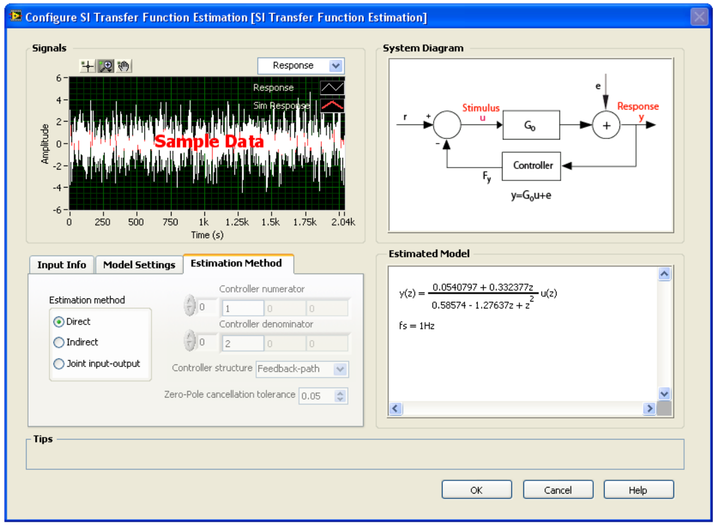


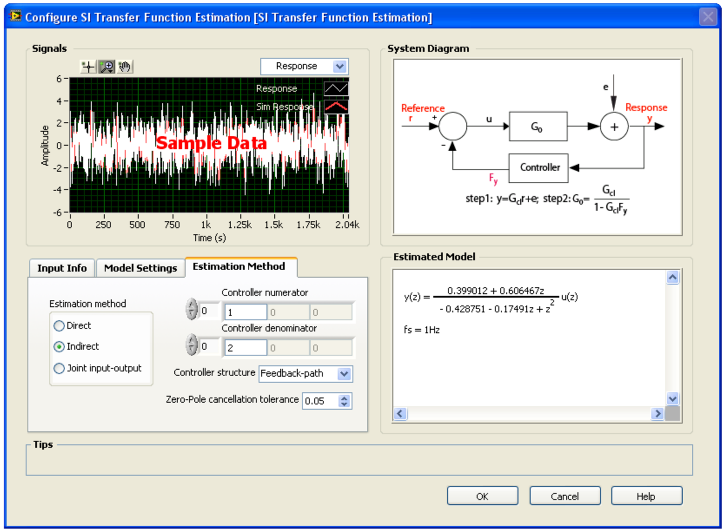
Closed Loop Systems

Table 6-1. Required Information for Closed-Loop Identification Approaches

	Stimulus Signal	Response Signal	Reference Signal	Controller Information
Direct	✓	✓	—	—
Indirect	—	✓	✓	✓
Joint Input-Output	✓	✓	✓	—

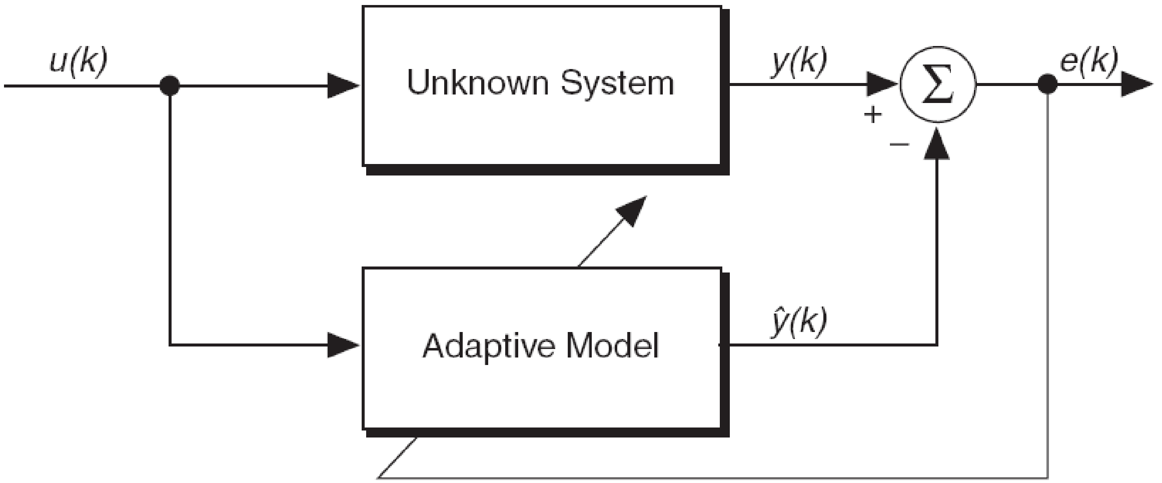






Recursive Identification

- Online Algorithms to adapt parameters while plant is in operation.
- Applies to ARX, ARMAX, OE, BJ, and GL models
- Can Choose from the following adaptive schemes
  - Least Mean Squares (LMS)
  - Normalized Least Mean Squares (NLMS)
  - Recursive Least Squares (RLS)
  - Kalman Filter

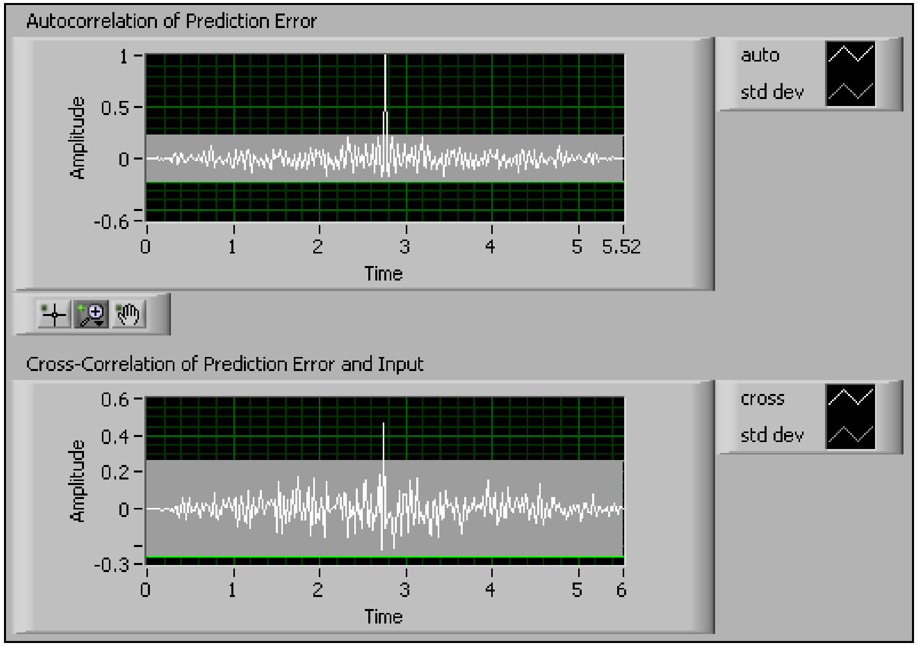


Analysis and Validation of Models

- Am I done?
  - Can the model predict the output for different types of input (imp, step, ramp, sine)?
  - Is the system stable? Plot the poles and zeros to observe stability.
  - Does the Bode look differences than systems' spectra from spectral analysis (GM, PM, resonances, bandwidth)
  - How does my model change with different frequency inputs? Examine the Nyquist plot (GM, PM)
- All these tools are available in LabVIEW

Analysis and Validation of Models

- Examine Autocorrelation of error residuals (with themselves)
  - Tells you whether the residuals are zero-mean white noise. If this is not true, then the model structure is not relevant to the system
- Cross Correlation (residuals and input)
  - Tells you if there is dynamics in the error indicating that you left them out of your model; may have to change model parameters



Conclusions

- There exists a tremendous amount of tools at your disposal.
- Nearly every tool has been automated and is in your version of LabVIEW.
- System Identification is a Design Problem that requires physical insight.
- Data Collection is the most important step
- Non-parametric Methods can provide insight into the system
- Families of Parametric models exist to capture nearly all types of systems (LINEAR)
- Parameters can be identified as well as the model structure
- Models must be validated.
- There is no tool that replaces your understanding of the system